Cr 53028

Contributed by the Heat Transfer Division for presentation at the Winter Annual Meeting, New York, N. Y., November 26-December 1, 1961, of *The American Society of Mechanical Engineers*. Manuscript received at ASME Headquarters, July 28, 1961.

N 64-80402

Code none

**Technical Release No. 34-227**

# Thermal Efficiency of Coated Fins

**J. A. Plamondon** Nov. 1961 4p refs

Presented at Am. Soc. of Mech. Engr. Winter Ann. Meeting, New York, 26 Nov. - 1 Dec. 1961

Conf

This paper presents results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract ~~No.~~ NASw-6, sponsored by the National Aeronautics and Space Administration.

2

(NASA Contract NASw-6)

(NASA CR-53028; JPL-TR-34-227; ASME Paper 61-WA-168)

JET PROPULSION LAB~~ORATORY~~
CALIF~~ORNIA~~ INST~~ITUTE~~ OF TECH~~NOLOGY~~
PASADENA ~~CALIFORNIA~~

4742003

~~November 1961~~

**Paper No.**
**61—WA-168**

**J. A. PLAMONDON**
Research Engineer,
Jet Propulsion Laboratory,
California Institute of Technology,
Pasadena, Calif.

# Thermal Efficiency of Coated Fins[1]

*The surfaces of a fin from which heat is rejected solely by radiation may have to be coated to obtain high values of thermal emissivity. In order to determine the influence of the conductive resistance of a coating on the thermal performance of a fin, an analysis was undertaken. Two equations are derived to describe the heat-transfer characteristics of a coated fin: One, a differential equation for the temperature profile on the radiating surfaces of the coating; and two, an equation for the relative thermal performance of the fin in terms of fin efficiency. The equations are solved numerically, and the fin efficiencies are plotted as a function of two dimensionless parameters which appear in the differential equation. These efficiencies are compared with those for fins in which the conductive resistance of the coating is ignored.*

## Introduction

In designing a radiator for utilization aboard a space vehicle, a primary objective is the maximization of heat rejected per unit weight [1].[2] A well-established, practical means for increasing the heat-transfer capability of a radiator is the incorporation of extended surfaces or fins. In space, where heat is rejected solely by radiation, the fin properties of thermal conductivity, thermal emissivity, and material density are of prime importance. It is unfortunate that materials having the desirable properties of high conductivity and low density tend to have poor emissivity.

To bypass the handicap of low emissivity, it has been suggested that fin surfaces be "blackened" by applying thin coatings of some suitable material. It has been assumed, and rightfully so, that if the coating is kept very thin, fin heat-transfer rates will not be inhibited by the coating. Using this assumption, one-dimensional studies of simple rectangular fins have been made. However, there are factors which may affect the feasibility of utilizing extremely thin coatings. First, in order to realize the full emissive power of a coating, the coating thickness must be greater than the optical penetration depth of the coating. Second, because of the hard vacuum of space, surface evaporation will occur. Finally, bombardment by micrometeorites will result in erosion. The latter two environmental factors may dictate relatively thick coatings to insure radiator longevity. If thick coatings are required, a substantial temperature drop through the coating may arise, thus inhibiting the heat-transfer capability of the fin. It is, therefore, the purpose of this paper to investigate the degree of inhibition imposed by the conductive resistance of a coating on the heat-transfer performance of a coated fin.

[1] This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract NASw-6, sponsored by the National Aeronautics and Space Administration.

[2] Numbers in brackets designate References at end of paper.

Contributed by the Heat Transfer Division for presentation at the Winter Annual Meeting, New York, N. Y., November 26–December 1, 1961, of The American Society of Mechanical Engineers. Manuscript received at ASME Headquarters, July 28, 1961. Paper No. 61—WA-168.

## Mathematical Formulation

Fig. 1 is a schematic diagram of a tube-fin radiator, showing the geometrical relationship of the fin which will be analyzed in the radiator design. It is assumed that the temperature of the tubes, and therefore the heat-transfer rate, is constant as a result of a fluid condensing at a constant rate in the tubes. A one-dimensional approach is taken in analyzing the fin, since it is felt that a one-dimensional solution will provide sufficiently accurate results while avoiding the complicating features of a more rigorous attack. The usual assumption for one-dimensional analyses of fins is made; namely, the fin thickness to length ratio is small, allowing one to assume that the temperature across the thickness of the fin is uniform. The following additional assumptions are also made: (*a*) Heat transfer to the surroundings is solely by radiation; (*b*) the radiating surface or surfaces have a geometrical view factor of one to the surroundings; (*c*) the surroundings are at 0 deg R; and (*d*) the thermal properties of both fin and coating are constant. Fig. 2 is a mathematical model to be used in the analysis of the fin.

In order to attack the problem one dimensionally, it is necessary to make one basic assumption in addition to those previously cited. It is necessary to assume that the temperature gradient in the coating in the $x$-direction is very small compared to the temperature gradient through the coating normal to the fin. Therefore, only that component of heat flow in the coating normal to the fin is considered. Implicit in this assumption is

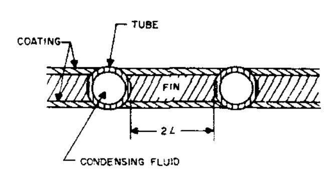


**Fig. 1 Schematic of a fin-tube radiator**

## Nomenclature

$k$ = thermal conductivity

$l$ = thickness

$L$ = length of fin or $1/2$ intertube spacing

$q$ = heat-transfer rate per unit length of tube

$T$ = temperature of fin at any point $x$, absolute units

$t$ = temperature on radiating surface of coating at any point $x$, absolute units

$x$ = distance co-ordinate

$\epsilon$ = thermal emissivity of radiating surface

$\theta$ = dimensionless temperature $t/t_0$

$\xi$ = dimensionless length $x/L$

$\eta$ = fin efficiency

**Subscripts**

0 = conditions of either the fin or the coating at $x = 0$

1 = fin

2 = coating



**Discussion on this paper will be accepted at ASME Headquarters until January 10, 1962**

the requirement that all the heat flowing in the $x$-direction is via the fin. This assumption introduces negligible error, if $(k_1l_1/L)/(k_2l_2/L) >> 1$ and $k_1/k_2 >> 1$. Offhand, these inequalities may appear rather restrictive, but in reality they are not, since materials presently considered suitable for fins and coatings for application in space, in general, satisfy the requirements.

**Temperature Profile on the Surface of the Coating.** Starting with the basic principle of energy conservation, Fig. 2 shows that

$$dq_{\text{cond}} + dq_{\text{rad}} = 0 \tag{1}$$

Evaluating the conductive term in the usual manner using Fourier's Law, for a unit width of fin,

$$\frac{d}{dx}\left[k_1 l_1 \frac{dT}{dx}\right] dx + dq_{\text{rad}} = 0 \tag{2}$$

Now, consider the heat flow through the coating $dq_{\text{rad}}$, in the elemental strip $dx$. Because it was assumed that the $x$-component of heat flow in the coating is negligible and that the temperature across the thickness of the fin is constant at any given location $x$, $dq_{\text{rad}}$ can be expressed by the simple relationships

$$dq_{\text{rad}} = -\frac{Bk_2}{l_2}(T - t)\, dx = -B\epsilon_2\sigma t^4 dx \tag{3}$$

These relationships simply state that in the elemental strip $dx$ the heat conducted through the coating is equal to the heat radiated from its surface. The constant $B$ is unity if heat is dissipated from only one side of the radiator with the other side insulated, and $B$ is two if heat is dissipated from both sides of the radiator.

By substituting the term at the right of equation (3) into equation (2) and simplifying,

$$\frac{d^2T}{dx^2} - \frac{B\epsilon_2\sigma t^4}{k_1l_1} = 0 \tag{4}$$

Equation (4) is not in useful form, since it contains two different, but related, temperatures $T$ and $t$. The relationship between $T$ and $t$ is found in the last two terms of equation (3) and is

$$T = \frac{l_2\epsilon_2\sigma t^4}{k_2} + t \tag{5}$$

Since $T$ is a function of $x$, the quantity $[(l_2\epsilon_2\sigma t^4)/k_2) + t]$ must also be a function of $x$; furthermore, it must be the same function of $x$ as $T$. Using this fact, equation (5) may be differentiated with respect to $x$. Differentiating twice yields

$$\frac{d^2T}{dx^2} = \frac{12l_2\epsilon_2\sigma t^2}{k_2}\left(\frac{dt}{dx}\right)^2 + \left(\frac{4l_2\epsilon_2\sigma t^3}{k_2} + 1\right)\frac{d^2t}{dx^2} \tag{6}$$

Substituting equation (6) into equation (4) and rearranging terms,

$$\frac{d^2t}{dx^2} + \frac{3}{\left(t + \dfrac{k_2}{4l_2\epsilon_2\sigma t^2}\right)}\left(\frac{dt}{dx}\right)^2 - \frac{Bt^2}{\dfrac{4k_1l_1l_2}{k_2}\left(t + \dfrac{k_2}{4l_2\epsilon_2\sigma t^2}\right)} = 0 \tag{7}$$

In order to write equation (7) in dimensionless form, let

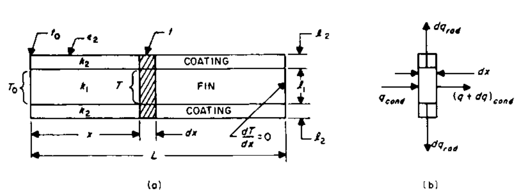


**Fig. 2 Mathematical model of fin**

$$x = \xi L \tag{8a}$$

and

$$t = \theta t_0 \tag{8b}$$

Upon substitution equation (7) becomes

$$\frac{d^2\theta}{d\xi^2} + \frac{3\theta^2}{\left(\theta^3 + \dfrac{k_2}{4l_2\epsilon_2\sigma t_0^3}\right)}\left(\frac{d\theta}{d\xi}\right)^2 - \frac{B\theta^4}{\dfrac{4k_1l_1l_2}{k_2L^2}\left(\theta^3 + \dfrac{k_2}{4l_2\epsilon_2\sigma t_0^3}\right)} = 0 \tag{9}$$

Here, the physical and thermal properties of the fin and coating are grouped into two dimensionless parameters which, for convenience, will be designated as $M$ and $N$, where

$$M = \frac{4k_1l_1l_2}{k_2L^2} \tag{10a}$$

and

$$N = \frac{k_2}{4l_2\epsilon_2\sigma t_0^3} \tag{10b}$$

Examination of these parameters reveals that they possess the following physical significance: When $M$ is written in the form

$$\frac{\dfrac{4k_1l_1}{L}}{\dfrac{k_2L}{l_2}}$$

$M$ is seen to be the ratio of the $x$-component of conductance in the fin to the normal component of conductance in the coating. Rewriting $N$ as

$$\frac{\dfrac{k_2L}{l_2}}{4L\epsilon_2\sigma t_0^3}$$

it is evident that $N$ is the ratio of the normal component of conductance in the coating to the emittance of the coating surface. By substituting $M$ and $N$ into equation (9), the final form of the differential equation is derived:

$$\frac{d^2\theta}{d\xi^2} + \frac{3\theta^2}{(\theta^3 + N)}\left(\frac{d\theta}{d\xi}\right)^2 - \frac{B\theta^4}{M(\theta^3 + N)} = 0 \tag{11}$$

The following boundary conditions conclude the mathematical formulation for the temperature profile along the radiating surface of the coating at

$$\xi = 0, \qquad \theta = 1 \tag{12a}$$

and at

$$\xi = 1, \qquad \frac{d\theta}{d\xi} = 0 \tag{12b}$$

**Fin Efficiency.** To complete the thermal description of a coated fin, a quantity is derived utilizing the temperature profile from the solution of equation (11) by which the heat-transfer rate of the fin can be readily computed. This quantity is also useful in judging the relative performance of a fin. It is generally referred to as "fin efficiency" and is defined as the ratio of the actual heat radiated $q_{\text{rad}}$ to that heat which would be radiated $q_0$ if the entire radiating surface were at the fin root temperature $T_0$; that is,

$$\eta \equiv \frac{q_{\text{rad}}}{q_0} \tag{13}$$

In the definition of $q_0$, no physical significance is implied. It is defined in such a manner simply to establish a datum or reference heat-transfer rate.

The actual heat radiated is obtained, using the temperature profile furnished by the solution of equation (11) and Stefan's Law, by

$$q_{\text{rad}} = B\,\epsilon_2\sigma \int_0^L t^4(x)\,dx \tag{14}$$

By definition, $q_0$ must be

$$q_0 = B\epsilon_2\sigma L T_0^4 \tag{15}$$

Regressing, at $x = 0$, equation (5) becomes

$$T_0 = \left(\frac{4l_2\epsilon_2\sigma t_0^3}{4k_2} + 1\right) t_0 \tag{16a}$$

but

$$\frac{1}{N} = \frac{4l_2\epsilon_2\sigma t_0^3}{k_2}$$

Thus

$$T_0 = \frac{4N + 1}{4N}\,t_0 \tag{16b}$$

Substituting equations (14), (15), and (16b) into equation (13) and nondimensionalizing using equations (8a) and (8b), the expression for fin efficiency becomes

$$\eta = \left(\frac{4N}{4N + 1}\right)^4 \int_0^1 \theta^4(\xi)d\xi \tag{17}$$

where $\theta(\xi)$ is the dimensionless-temperature profile furnished by the solution of equation (11), (12a), and (12b). In optimizing or maximizing a finned radiator design with respect to some criterion, such as weight or area, it turns out that fin efficiency is a very useful quantity.

## The Noninsulating Coating

To determine the degree to which a coating inhibits the heat-transfer capability of a coated fin, consider the mathematical formulation for a coated fin in which the conductive resistance of the coating is assumed to be negligible and thus produces no temperature drop. Such a situation will be referred to as "noninsulating." If the coating thickness $l_2$ is allowed to approach zero, mathematically, one is essentially considering the latter situation. When $l_2$ goes to zero, equation (11) reduces to

$$\frac{d^2\theta}{d\xi^2} - \frac{L^2\epsilon_2\sigma T_0^3}{k_1l_1} B\theta^4 = 0 \tag{18}$$

where $T = t$; thus $\theta = T/T_0$. As a consequence, the expression for fin efficiency reduces to

$$\eta = \int_0^1 \theta^4(\xi)d\xi \tag{19}$$

Note that equation (18) contains only one dimensionless parameter, which will be designated by $R$; that is,

$$R = \frac{L^2\epsilon_2\sigma T_0^3}{k_1l_1} \tag{20}$$

Physically, this parameter represents the ratio of radiant emittance of the coating to the $x$-component of fin conductance; i.e.,

$$\frac{L\epsilon_2\sigma T_0^3}{\dfrac{k_1l_1}{L}}$$

By comparing the solutions of equation (11) and (17) with those of equations (18) and (19), the degree to which the conductive resistance of the coating affects the heat-transfer rate of the fin can be evaluated. However, to facilitate the comparison, it is necessary to obtain a parameter which will link the solutions. If one considers the quotient $1/MN$, one finds that it has the same physical significance as $R$. However, $1/MN$ is not exactly $R$, since $t_0$ appears in the quotient rather than $T_0$. If $t_0$ is transformed to $T_0$ in $1/MN$ or $T_0$ is transformed to $t_0$ in $R$, the parameters become identities and the necessary link is obtained. In $1/MN$, $t_0$ will be transformed to $T_0$, since in the specifications of a radiator, the fin root temperature $T_0$ will be stated, whereas $t_0$ must be calculated. Since $t_0$ appears only in the parameter $N$, it is necessary to transform only $N$. This new parameter shall be denoted by $N'$, where $N'$ is related to $N$ by

$$N' = \frac{(4N + 1)^3}{64N^4} = \frac{4l_2\epsilon_2\sigma T_0^3}{k_2} \tag{21}$$

The transformation was made using equation (16b). Dividing $N'$ by $M$, yields $R$:

$$\frac{N'}{M} = \frac{4l_2\epsilon_2\sigma T_0^3}{k_2}\,\frac{k_2L^2}{4k_1l_1l_2} = \frac{L^2\epsilon_2\sigma T_0^3}{k_1l_1} = R \tag{22}$$

It should be pointed out that equations (18) and (19) are also applicable to uncoated fins by simply replacing the emissivity of the coating $\epsilon_2$ with the emissivity of the fin $\epsilon_1$.

## Solutions

Numerical solutions of the equations for coated fins [equations (11) and (17)] were obtained for a range of values of the parameters $M$ and $N$, using an IBM 704 computer. The values for $M$ and $N$ were computed based on probable values of fin and coating

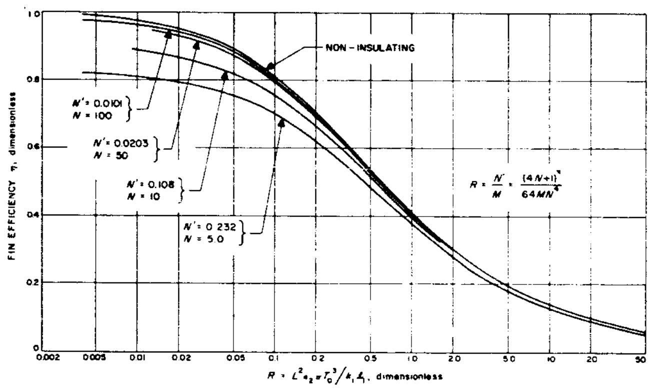


**Fig. 3 Fin efficiency as a function of R for constant parameter of M**

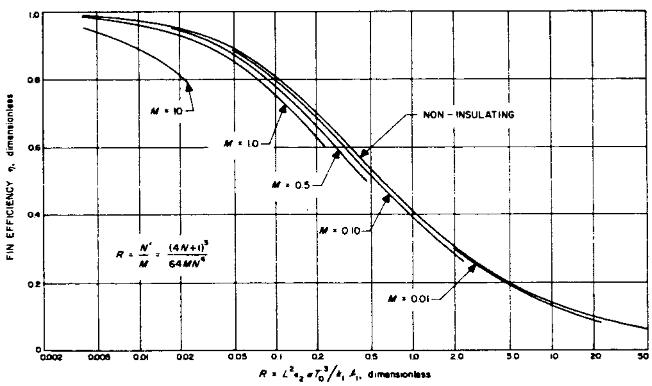


**Fig. 4 Fin efficiency as a function of R for constant parameter of N**

thermal and physical properties. The fin efficiencies obtained from equation (17) were plotted against the quotient $N'/M$ or $R$ for the following: (*a*) constant values of $N$ or $N'$ and variable values of $M$ (Fig. 3); and (*b*) constant values of $M$ and variable values of $N$ or $N'$ (Fig. 4).

Numerical solutions for the noninsulating case [equations (18) and (19)] are available in the literature for fin efficiency as a function of $R$ [2, 3, 4]. These solutions are also plotted in Figs. 3 and 4 and are denoted by the term noninsulating. For both cases, a value of two was chosen for $B$.

## Discussion

The effect of the conductive resistance to the flow of heat imposed by a coating has been obtained by considering its effect on fin efficiency. This effect is clearly illustrated in Figs. 3 and 4 by the difference between the curves described by the parameters $M$ and $N'$ (or $N$) and the curve labeled noninsulating.

The difference, and thus the degree to which fin efficiency is affected, increases as either or both parameters $M$ or $N'$ increase. In ascertaining the limits to which these parameters can increase, it was found that for a typically designed, coated fin, $M$ would probably be less than two or three. On the other hand, the maximum value of $N'$ appears to be limited only by the maximum root temperature $T_0$ which the fin and/or coating can withstand. For an aluminum fin, this maximum root temperature is roughly 1200 deg R. Using less desirable materials, higher temperatures are feasible but not necessarily optimal from the standpoint of obtaining reasonable fin efficiencies. Note that as $T_0$ increases, the parameter $R$ also increases, resulting in lower fin efficiencies. It should be made clear, however, that an increase in $T_0$ results in a substantial increase in the heat-transfer rate even though fin efficiency decreases. To compensate for the deleterious effect of increasing temperature, it is necessary to shorten the fin. As the length of the fin decreases, the parameter $M$ increases and thus the negative effect imposed by the conductive resistance of the coating increases. When $T_0$ reaches relatively high values, resulting in rather low-coated fin efficiencies, it is no longer possible to meet the objective of maximum heat rejected per unit weight using a finned radiator design. Studies show that a crossover point is reached as $T_0$ increases at which, on a weight basis, it is more desirable to radiate the heat directly from the surfaces of the tubes, since the fins become ineffectual [4, 5]. The effect of the coating is to lower the temperature level at which the crossover point is reached.

The assumption that the coating imposes negligible conductive resistance to the flow of heat is reasonable if $M < 0.1$ and $N'$ is relatively small. For example, if $N' < 0.0203$ and $M < 0.1$, the conductive resistance of the coating affects fin efficiency by less than 3 per cent; on the other hand, if $N' > 0.108$, then fin efficiency is affected by 6 per cent or more.

It may be advantageous to study an example of what, perhaps, can be referred to as a typical coated fin. Consider first an aluminum fin having a relatively thin coating of aluminum phosphate. The physical dimensions and thermal properties of the fin and coating are shown in Table 1.

**Table 1 Values for a typical coated fin**

| Thermal properties | | Dimensions, ft | |
|---|---|---|---|
| $k_1$, Btu/hr ft deg R | 140 | $l_1$ | 0.003 |
| $k_2$, Btu/hr ft deg R | 0.2 | $l_2$ | 0.0003 |
| $\epsilon_2$ | 0.9 | L | 0.1 |

Assume that the fin root temperature is 1200 deg R. The computed values of $R$, $M$, and $N'$ are, respectively, 0.064, 0.252, and 0.0162. From Figs. 3 or 4, the fin efficiency, if the coating is noninsulating, is found to be 0.87. If the conductive resistance of the coating is included, the fin efficiency is 0.85. The effect of including the conductance of the coating results in a 2.3 per cent decrease in fin efficiency. Now assume that the coating must be relatively thick as a result of any or all of the factors cited in the introduction. Assume that $l_2$ is increased to $15 \times 10^{-4}$ ft. The values of $R$, $M$, and $N'$ are, respectively, 0.064, 1.26, and 0.0806. The efficiency for the noninsulating case remains 0.87; for the insulating case, it becomes 0.80. The conductive resistance of the coating now produces an 8.1 per cent loss in efficiency. In the final design analysis of a finned radiator, for use in space, the neglect of such a loss will not be tolerable. Any property that increases either $M$ or $N'$, or both, will similarly affect fin efficiency.

In concluding, it should be pointed out that when studying the influence of a property on fin efficiency, it is helpful to consider first the influence of the property on the parameter in which it appears ($R$, $M$, or $N'$) in terms of the parameter's physical significance. For example, if the emissivity in the parameter $R$ is increased, the fin efficiency decreases, which appears, at first glance, contrary to what one might expect. However, if the physical significance of $R$ is considered, one notes that an increase in emissivity results in an increase in radiant emittance of the coating, while fin conductance remains unchanged. The surface is, therefore, capable of radiating relatively more heat than the fin is able to conduct. Thus, by the definition of fin efficiency, the efficiency must decrease. Nevertheless, an increase in emissivity is desirable, since the heat-transfer rate increases even though fin efficiency decreases.

## References


1 J. P. Callinan and W. P. Berggren, "Some Radiator Design Criteria for Space Vehicles," Journal of Heat Transfer, Trans. ASME, Series C, vol. 81, 1959, pp. 237–244.

2 J. G. Bartas and W. A. Sellers, "Radiation Fin Effectiveness," Journal of Heat Transfer, Trans. ASME, Series C, vol. 82, 1960, pp. 73–75.

3 S. Lieblein, "Analysis of Temperature Distribution and Radiant Heat Transfer Along a Rectangular Fin of Constant Thickness," NASA TN D-196, 1959.

4. M. G. Coombs, R. A. Stone, and T. Kapus, "The Snap 2 Radiative Condenser Analysis," Atomics International, Division of North American Aviation, Inc., NAA-SR-5317.

5. R. E. English, H. O. Slone, D. T. Bernatowicz, E. H. Davison, and S. Lieblein, "A Twenty-Thousand Kilowatt Nuclear Turboelectric Supply for Manned Space Vehicle," NASA Memo 2-20-59E, 1959.


Printed in U. S. A.